FRACTALS

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INTRODUCTION

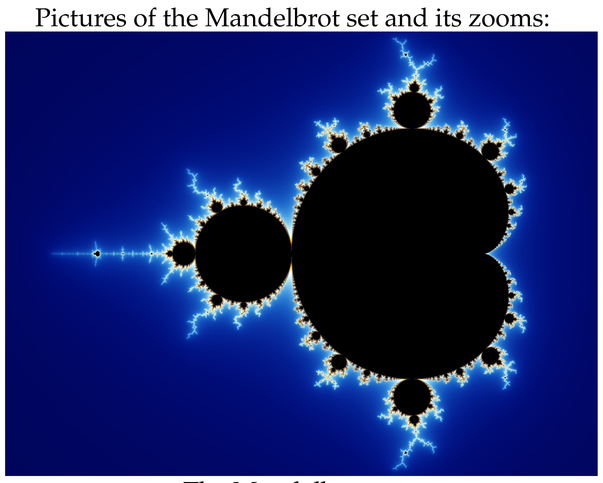
Roughly speaking a **fractal** is a set which presents a self-similar pattern (the same pattern repeated in every scale) or quasi-self-similar pattern (a fractal is almost the same in every scale). The pattern also has to be detailed. The name fractal was introduced by Benoit Mandelbrot. In fact, there is some controversy about the formal definition of a fractal but let us see some examples and their properties.

EXAMPLES

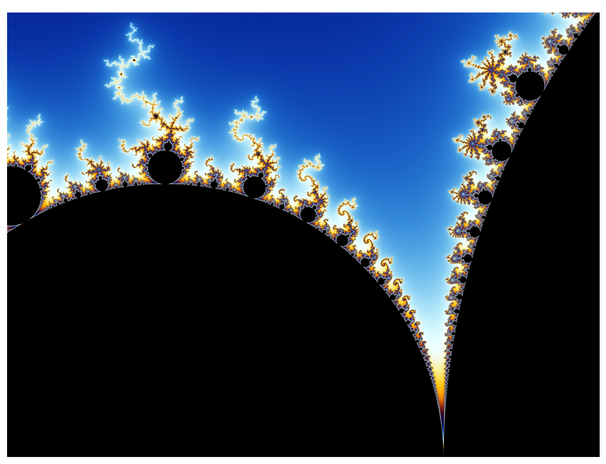
Mandelbrot set

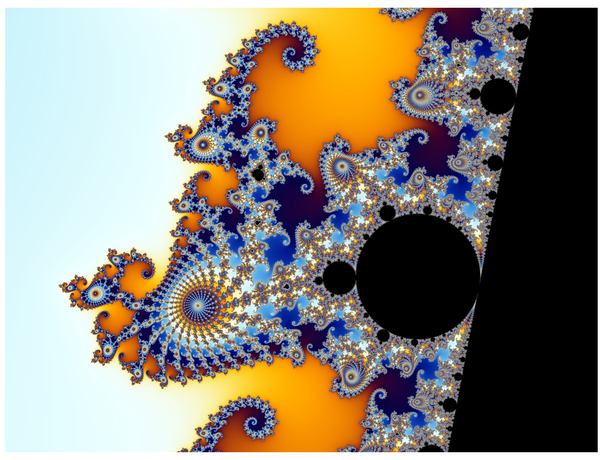
Before defining the Mandelbrot set we quickly introduce the notion of a complex number. Define i to be a number such that $i^2 = -1$. A complex number is a number which is of the form $a + bi$ where a and b are some real numbers. We are multipling complex numbers in the following way: $(a + bi)(c + di) = ac - bd + (ad + bd)i$.
The Mandelbrot set is the set of those complex numbers z such that the sequence $(z, z + z^2, z + (z + z^2)^2, \cdots)$ is bounded. In the picture of the Mandelbrot set, the color of a point z shows how fast the sequence $(z, z + z^2, z + (z + z^2)^2, \cdots)$ diverges (if this is the case). The Mandelbrot set has the property of the quasi-self-similarity.

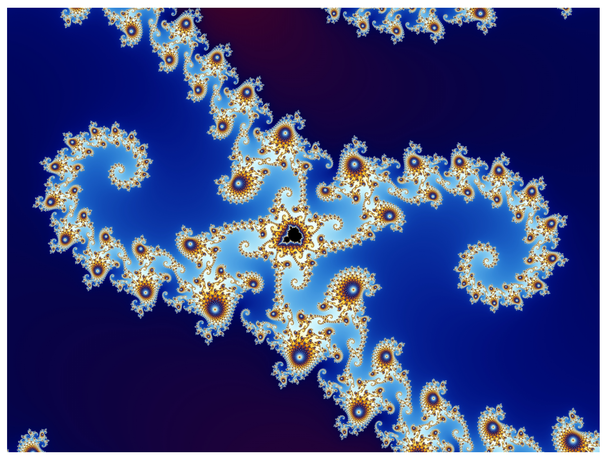
Pictures of the Mandelbrot set and its zooms:

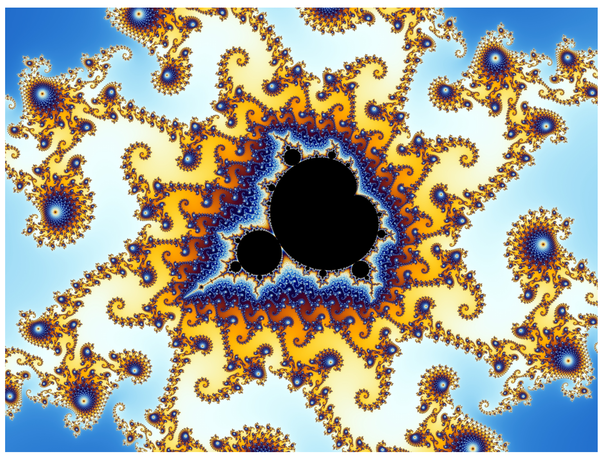


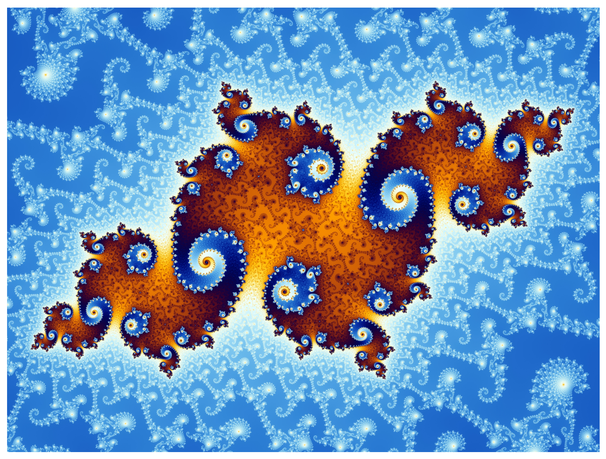
The Mandelbrot set





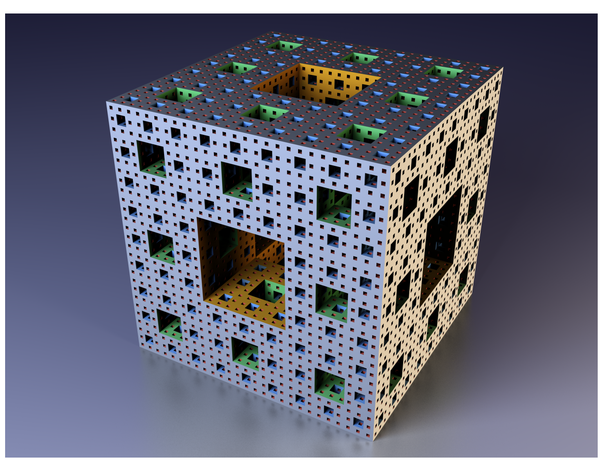




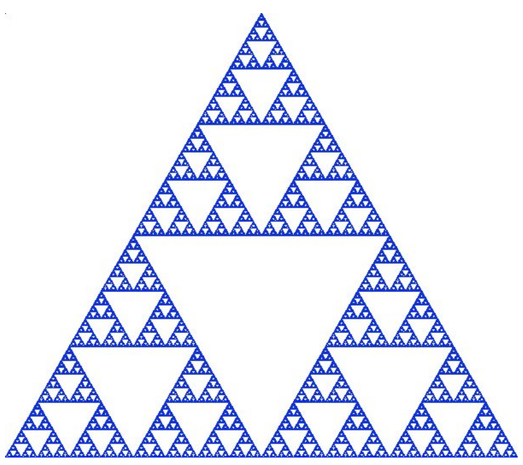


Menger sponge and Sierpinski triangle

Menger sponge and Sierpinski carpet are examples of a fractal with an exact self-similar pattern.



Menger sponge



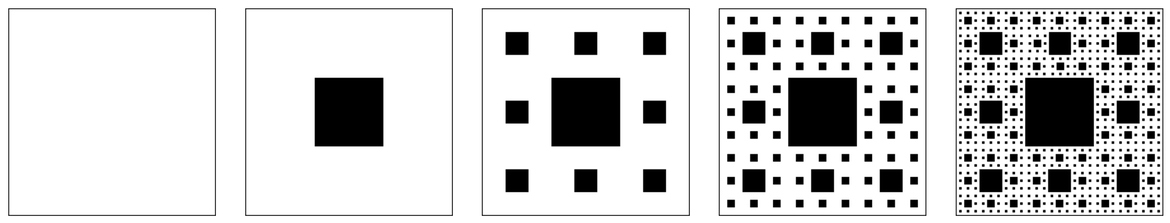
Sierpinski triangle

CONSTRUCTION

Some of fractals could be constructed using an iterated function.

Sierpinski carpet

To construct Sierpinski carpet let us take a square and divide into 9 identical subsquares and remove the central one. We repeat the procedure for the remaining 8 squares and so on. Here are the first steps of the construction:



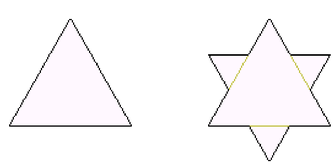
Remark: Menger sponge is a 3-dimesional generalistion of Sierpinski carpet.

Koch snowflake

To construct the Koch snowflake take an equilateral triangle and then:

- divide each side into three segments of the same length
- consider equilateral triangles having the middle segment from the previous step as one of their sides
- remove the middle segments from the previous step.

Then repeat this three steps infinitely many times.



First steps in construction of the Koch snowflake

FRACTALS IN NATURE

In nature there are some structures with approximate self-similar pattern which repeats finitely many times.

Romanesco broccoli

"Lichtenberg figure"

FRACTALS IN ART

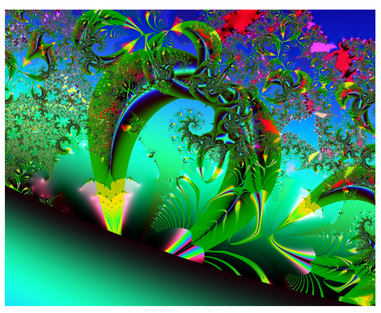


image done in the program Sterling

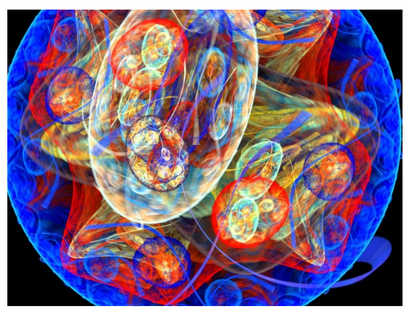


image done in the program Electric Sheep

Bibliography:

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Pictures

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"Mandel zoom 01 head and shoulder", "Mandel zoom 03 seehorse", "Mandel zoom 06 double hook", "Mandel zoom 07 satellite", "Mandel zoom 14 satellite julia island", Wolfgang Beyer
"Menger-Schwamm-farbig" by Niabot - Own work, http://commons.wikimedia.org/wiki/File:Menger-Schwamm-farbig.png#/media/File:Menger-Schwamm-farbig.png
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"Menger 1", "Menger 2", "Menger 3", "Menger 4"Arbol01
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"Fractal Broccoli" by Jon Sullivan - http://pdphoto.org/PictureDetail.php?mat=pdef&pg=8232 http://commons.wikimedia.org/wiki/File:Fractal_Broccoli.jpg#/media/File:Fractal_Broccoli.jpg
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